Sec 8.1 Variation Functions page 573

Given: \( y \) varies directly as \( x \). Find the constant of variation, \( k \), and write the variation function.

2. \( y = 6 \) when \( x = 3 \)
   \[
   y = kx \\
   6 = 3k \\
   k = 2 \\
   y = 2x
   \]

3. \( y = 45 \) when \( x = -5 \)
   \[
   y = kx \\
   45 = 5k \\
   k = 9 \\
   y = -9x
   \]

4. \( y = 54 \) when \( x = 4.5 \)
   \[
   y = kx \\
   54 = 4.5k \\
   k = 12 \\
   y = 12x
   \]

5. The wavelength \( \lambda \) of a wave of a certain frequency varies directly as the velocity \( v \), of the wave, and \( \lambda = 60 \) feet when \( v = 15 \) ft/s. Find \( \lambda \) when \( v = 3 \) ft/s.
   \[
   \lambda = k v \\
   60 = k(15) \\
   k = 4 \\
   \lambda = 4(3) \\
   \lambda = 12 \text{ feet}
   \]

6. The dollar amount \( d \) that Julia earns varies directly as the number of hours \( t \) that she works, and \( d = 116.25 \) when \( t = 15 \). Find \( t \) when \( d = 178.25 \).
   \[
   d = k(t) \\
   116.25 = k(15) \\
   k = 7.75 \\
   \frac{178.25}{7.75} = k(15)
   \]

7. The volume \( V \) of a rectangular prism of a particular height varies jointly as the length \( l \) and the width, \( w \), and \( V = 224 \) ft cubed when \( l = 8 \) ft and \( w = 4 \) ft. Find \( l \) when \( V = 210 \) ft. cubed and \( w = 5 \) ft.
   \[
   V = k(lw) \\
   224 = k(8)(4) \\
   k = 7 \\
   V = 210 = (7x5)l \\
   210 = 35l \\
   l = 6
   \]

Given: \( y \) varies inversely as \( x \). Find the constant of variation \( k \), and write the variation function.

9. \( y = 2 \) when \( x = 7 \)
   \[
   y = \frac{k}{x} \\
   2 = \frac{k}{7} \\
   k = 14 \\
   y = \frac{14}{x}
   \]

10. \( y = 8 \) when \( x = 4 \)
    \[
    y = \frac{k}{x} \\
    8 = \frac{k}{4} \\
    k = 32 \\
    y = \frac{32}{x}
    \]

11. \( y = \frac{1}{2} \) when \( x = -10 \)
    \[
    y = \frac{k}{x} \\
    \frac{1}{2} = \frac{k}{-10} \\
    k = -5 \\
    y = \frac{-5}{x}
    \]

12. The time \( t \) that it takes for a salesman to drive a certain distance \( d \) varies inversely as the average speed \( r \). It takes the salesman 4.75h to travel between two cities at 60 mi/h. How long would the drive take at 50 mi/h?
    \[
    t = \frac{k}{d} \\
    4.75 = \frac{k}{60} \\
    t = 5.7 \text{ hours}
    \]
    \[
    k = 285
    \]
27. The number of days it takes a movie crew to set up a stage for a scene varies inversely as the number of workers. If the stage can be set up in 3 days by 20 workers, how many days would it take if only 12 workers were available?

\[
d = \frac{k}{w} \Rightarrow \frac{20}{3} = \frac{k}{20} \Rightarrow k = 400 \text{ workers-days}
\]

\[
d = \frac{400}{12} = 33 \frac{1}{3} \text{ days}
\]

Determine whether each data set represents a direct variation, an inverse variation, or neither.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>x</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>y</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
y \text{ varies directly as } x, \text{ and } y = 30 \text{ when } x = -6.
\]

\[
y = \frac{kx}{5}
\]

\[
y = \frac{-5x}{3}
\]

Write and graph each variation function.

\[
y \text{ varies inversely as } x, \text{ and } y = 5 \text{ when } x = 3.
\]

\[
y = \frac{15}{x}
\]

\[
y = \frac{15}{x}
\]

Coming UP

Simplify each expression. Assume all variables are nonzero.

a) \(x^7 \cdot x^2 = x^{7+2} = x^9\)

b) \(y^3 \cdot y^3 = y^{3+3} = y^{6}\)

c) \(x^6 \cdot \frac{x^{10}}{x^2} = \frac{x^{6+10}}{x^2} = \frac{x^{16}}{x^2} = x^{14}\)

d) \(\frac{y^2}{y^5} \cdot y^2 - 5 = \frac{1}{y^3}\)

e) \(x^2 - 2x - 8 = (x-4)(x+2)\)

f) \(x^2 - 5x = x(x-5)\)

g) \(x^4 - 9x^3 = x^3(x^2 - 9) = x^3(x+3)(x-3)\)