Algebra 2 Worksheet  
Section 6.6 - Fundamental Theorem of Algebra DAY TWO

I. Write the simplest polynomial function (in standard form) with the following zeros.

1. 3i and -5
   \((x)^2 = (3i)^2\)
   \(x^2 = 9i^2\)
   \(x^2 = -9\)
   \(x^2 + 9 = 0\)

   \(p(x) = (x+5)(x^2 + 9)\)
   \(p(x) = x^3 + 5x^2 + 9x + 45\)

2. -2i and \(\sqrt{2}\)
   \((x)^2 = (-2i)^2\)
   \(x^2 = 4i^2\)
   \(x^2 = -4\)
   \(x^2 - 4 = 0\)
   \(x^3 + 2x^2 - 8\)

II. Solve each equation by finding all the roots (EXACT VALUE). Use the calculator to help you.

3. \(x^3 - 7x^2 + 15x - 9 = 0\)
   Roots @ -1 and 3
   Double root
   \(X = -1, 3\)

4. \(14x^3 - 43x^2 + 14x^2 + 15x = 0\) CAREFUL GCF!
   \(x(14x^3 - 43x^2 + 14x + 15) = 0\)
   0 is a root
   \(\frac{14}{-210}\)
   \(\frac{-29}{-29}\)
   \(\frac{-15}{-15}\)
   \(\frac{14}{14}\)
   \(\frac{-29}{-29}\)
   \(\frac{-15}{-15}\)
   \(0\)
   \(\frac{-210}{0}\)
   \(14x^2 - 29x - 15 = 0\)
   Multi: -210
   Add: -29
   -35 and 6
   \((2x - 5)(7x - 3) = 0\)
   \(2x - 5 = 0\)
   \(7x + 3 = 0\)
   \(x = \frac{5}{2}\)
   \(x = -\frac{3}{7}\)
   \(x = 0, 1, \frac{5}{2}, -\frac{3}{7}\)
5. \( x^4 - 3x^3 + 5x^2 - 27x - 36 = 0 \)
   \[
   \begin{array}{c|cccc}
   & 1 & -3 & 5 & -27 \\
   \hline
   1 & 1 & 2 & -9 \\
   \end{array}
   \]
   \[
   \begin{array}{c|ccccc}
   & -1 & 1 & 9 & -9 \\
   \hline
   1 & 1 & 9 & 0 \\
   \end{array}
   \]
   \[
   x^2 + 9 = 0 \\
   x = \pm 3i
   \]

6. \( x^3 - 2x^2 - 2x - 3 = 0 \)
   \[
   \begin{array}{c|cccc}
   & 1 & -2 & -2 & -3 \\
   \hline
   1 & 1 & 3 & 0 \\
   \end{array}
   \]
   \[
   \begin{array}{c|ccccc}
   & 1 & -2 & -2 & -3 \\
   \hline
   1 & 1 & 3 & 0 \\
   \end{array}
   \]

III. Graphing Calculator.

7. Consider the polynomial function \( f(x) = 3x^4 + 40x^3 + 96x^2 + 144x - 715 \).
   (a) Use the Rational Root Theorem to list the possible rational roots of this equation.
   \[
   \pm 1, \pm 5, \pm 11, \pm 13, \pm 55, \pm 65, \pm 143, \pm 715
   \]
   \[
   \pm 1, \pm 3
   \]
   \[
   \frac{a}{b} : \pm 1, \pm 5, \pm 11, \pm 13, \pm 55, \pm 65, \pm 143, \pm 715
   \]
   \[
   (b) \ Graph \ the \ polynomial \ on \ a \ graphing \ calculator. \ Which \ possible \ rational \ roots \ are \ zeros \ of \ \( f(x) \)? \ How \ do \ you \ know? \\
   \[
   \begin{array}{c|c}
   x \text{-int of function} & \frac{5}{3} \\
   \hline
   -11 & \frac{5}{3}
   \end{array}
   \]
   (c) According to the graph, how many other real zeros does the function have? 
   \[
   \text{None}
   \]
   (d) How many imaginary zeros does the function have?
   \[
   2
   \]
   \[
   (4 \ \text{total roots} \rightarrow 2 \ \text{real} \rightarrow 2 \ \text{imag})
   \]
   (e) Find the imaginary zeros. SHOW YOUR WORK.
   \[
   \begin{array}{c|ccccc}
   & 3 & 40 & 96 & 144 & -715 \\
   \hline
   1 & 3 & 7 & 19 & -65 \\
   \end{array}
   \]
   \[
   \begin{array}{c|ccccc}
   & 3 & 7 & 19 & -65 \\
   \hline
   1 & 3 & 7 & 19 & -65 \\
   \end{array}
   \]
   \[
   x^2 + 12x + 39 = 0 \\
   x = -4 \pm \frac{\sqrt{16 - 4(13)}}{2}
   \]
   \[
   x = -2 \pm 3i
   \]